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A localised, ensemble-based data assimilation for steady-state, laminar flow past a rectangular cylinder

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SUMMARY:
Numerical and experimental methods have been implemented independently for many years for wind engineering applications. Their combination, using data assimilation (DA), is a relatively recent development in the field in order to provide an improved solution. In this scope, a DA method is developed based on the employment of the Best Linear Unbiased Estimator (BLUE) equations combined with an ensemble-based strategy for the quantification of the underlying uncertainties in an explicitly localised framework. For a steady-state, laminar flow past a rectangular cylinder, synthetic measurements of the two velocity components and pressure, originating from a finer mesh CFD solution, are assimilated into a model state derived from a coarser mesh solution, using the same CFD code. The spatial discretisation step is considered as the only source of relative uncertainty between the two solutions and hence, it is perturbed for the generation of the ensemble statistics at a Reynolds number of $Re = 10$. These ensemble statistics are also used for DA at an order of magnitude range of Reynolds numbers belonging to a common flow regime. Significant model error reduction is accomplished, even with a limited number of assimilated measurements and even if they are derived at a different Reynolds number.

Keywords: Data Assimilation, Ensemble Kalman Filter, Computational Wind Engineering

1. INTRODUCTION

The common practice in the discipline of fluid mechanics consists of treating the CFD solutions and the measurement sets as independent sources of information even though they may quantify the same phenomenon. Both methods have specific advantages and disadvantages and thus, the combination of CFD and measurement results constitutes a significant advancement towards the acquisition of an improved final solution by capitalising on these advantages. Indicative data/measurement integration attempts (where the uncertainty is not usually quantified) can be found as early as e.g. Hayase and Hayashi (1997) and, more recently, Pallas and Bouris (2022) where measurements (synthetic or 2D PIV) are integrated into SIMPLE-based algorithms for the purpose of boundary condition or pressure field reconstruction for duct flow or the flow past a surface mounted cube, respectively. Data assimilation (where the uncertainty is usually quantified) has been a common practice for years in the field of meteorology (Bengtsson et al., 1981) also gaining increasing popularity in other disciplines, including fluid mechanics. Recently the Kalman Filter (KF) was used for wind power forecast in a micro-scale wind farm (Liu and Liang, 2021). A comparative study between variational, Ensemble KF-based and hybrid data assimilation (DA) methods was undertaken by Mons et al. (2016) for reconstruction of the flow past a cylinder in the presence of incident coherent gusts.

In the current attempt, the generation of ensembles is accomplished through perturbation of a purely numerical/mathematical parameter (i.e. the spatial discretisation step) of the problem, instead of considering that all the uncertainty originates from uncertain physics, unknown parameters or inexact boundary/initial conditions, as it is usually done (Resseguier et al., 2021). Explicit localisation (Hunt et al., 2007) is also performed in order to reduce the computational cost as well as overcome rank deficiency problems of the participating matrices. Finally, parametric analysis is performed for a range of Reynolds numbers (*Re*) using the same ensemble statistics acquired at a specific *Re*. To the best of the authors' knowledge, this is a novel perspective in the framework of the ensemble-based methods and a first step before extension to turbulent flow.

2. EQUATIONS AND METHODOLOGY

If $\overline{\boldsymbol{x_b^j}}$, $\boldsymbol{y}$ and $\overline{\boldsymbol{x_a^j}}$ are the model solution vector, the measurement solution vector, and the analysis (corrected) solution vector, respectively, Eq. (1) gives the correction procedure (Evensen, 2009). This may be applied to one or more of the variables involved in the solution, where H is a mapping matrix, K is the Kalman Gain and the overbar denotes ensemble-averaged quantities. The superscript j indicates a single ensemble member derived by Monte-Carlo (MC) simulations of a CFD code, which uses perturbed values of a chosen input parameter, sampled from a Gaussian distribution.

$$\overline{\boldsymbol{x_a^j}} = \overline{\boldsymbol{x_b^j}} + K(\boldsymbol{y} - H\overline{\boldsymbol{x_b^j}}) \quad \text{with} \quad K = P_b H^T (H P_b H^T + R)^{-1} \tag{1}$$

The Kalman gain determines the degree of correction through the term $K(\boldsymbol{y} - H\overline{\boldsymbol{x_b^j}})$, by taking into account the relative uncertainty between the two sources of information. This is clarified by its definition, shown in Eq. (1), since P_b and R are the model and error covariance matrices, respectively. Ensemble-based methods utilise Eq. (2) to quantify the model uncertainty via the model error covariance matrix, where $\boldsymbol{x_b^j}$ corresponds to a random model realisation and N_{ens} is the size of the ensemble.

$$P_b = \frac{1}{N_{ens}} \sum_{j=1}^{N_{ens}} [(\overline{\boldsymbol{x_b^j}} - \boldsymbol{x_b^j})(\overline{\boldsymbol{x_b^j}} - \boldsymbol{x_b^j})^T] \tag{2}$$

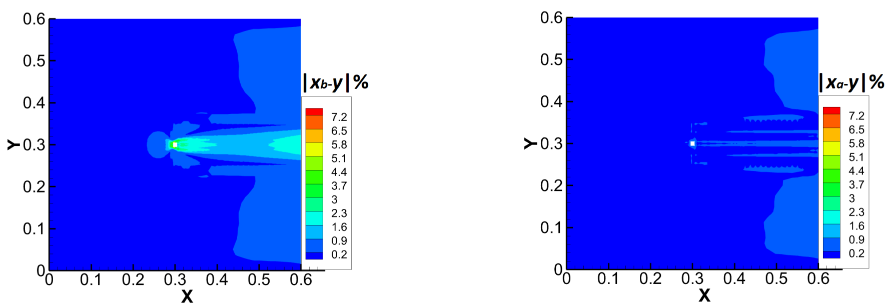
It is well known that the size of the ensemble limits the rank of the model error covariance matrix since it can be at most equal to $N_{ens} - 1$ (Evensen, 2009; Hunt et al., 2007) so when the number of model state variables n is significantly higher than $N_{ens} - 1$, the matrix inversion (in K) of Eq. (1) is rendered infeasible. Explicit localisation (Hunt et al., 2007), used here, provides a solution by omitting spatial correlations between state variables beyond a certain distance. Cross-variable correlations (e.g. among velocity components and pressure) are considered equal to zero for the case study of this paper, to alleviate further complexity.

3. RESULTS

The application presented here, is the case of 2D, steady-state, laminar flow past a square cylinder at $Re = 10$. The grid has an aspect ratio of 1 and $\Delta X_{ref,finer} = d/10$ for the finer mesh/"measurement set" while for the coarser mesh/"model solution" $\Delta X_{ref,coarser} = d/4$, with d being the edge of the rectangular cylinder. A square computational domain with dimension equal to $60d$ is employed.

The CFD solutions are extracted via the OpenFOAM utility, simpleFoam, based on the SIMPLE algorithm. Calculated C_D values and recirculation lengths pertaining to the finer mesh solution agree well ($\leq 5\%$) with those in the open literature. The ensemble members are created by considering uncertainty only in the spatial discretisation. This assumption is based on the fact that: (a) the same OpenFOAM code is utilised for the measurement set and the model solution, (b) a sensitivity analysis showed that the difference in the accuracy induced by different spatial resolutions of ΔX in the range $[0.008d, 0.4d]$, is reflected satisfactorily onto various output variables (e.g., the drag coefficient C_D) and (c) without turbulence modelling, there are no physical assumptions in the equations.

For the application of the method to the two velocity components and the pressure (i.e., u, v and p, respectively), the size of the ensemble and the distance of localisation were chosen after respective parametric analyses. The calculated l^2 error norms $||x_a - y||$ (the overbar and the superscript j are omitted) after the assimilation of the whole measurement set ($f = 100\%$), are found to be reduced by 80, 70 and 55% for the u,v and p variables, respectively. Elaborate diagnostic tools/methods of measurement space reduction were also applied to show that by carefully choosing the measurement space, impressive model error reduction can be achieved even for limited numbers of assimilated measurements, e.g. with 10% of the measurements the error reduction is 60%, for v. Indicative performance of the method is demonstrated through analysis errors in Fig. 1.



(a) (b)

Figure 1. Error contours for u between measurement set ($f = 100\%$) and: (a) model and (b) analysis solutions.

Results of the drag coefficient relative error $\varepsilon_{C_{D,a}} = |C_{D,a} - C_{D,y}|/|C_{D,y}|$ (a: analysis, y: measurements) after the implementation of DA also show significant model error reduction. The respective error value before DA is 5.58% while after DA for u, v and p ($f = 100\%$), it becomes 0.98%. If only "surface pressures" ($f = 0.08\%$) are assimilated along with all ($f = 100\%$) velocity measurements, ε_{C_D} is 1.24%, (i.e. only 25% higher than if all pressure measurements are used), indicating that it may be sufficient for a proper C_D calculation. For DA of only u and v measurements ($f = 100\%$ but without pressure) the error is much higher (1.97%).

The method is also implemented by utilising the same covariance matrices produced by ensemble members at $Re = 10$ for the assimilation of measurements in the range $Re \in [5, 50]$ i.e. an order of magnitude range of Re numbers, containing the whole steady-state, laminar regime (Jiang and Cheng, 2018). The reduction in the relative error $\varepsilon_r = 1 - ||x_a - y||/||x_b - y||$ quantifying the DA correction is plotted against Re number for u,v and p, in Fig. 2. The attained DA correction for all three variables is higher than 50% almost for the full span of the examined Re range.

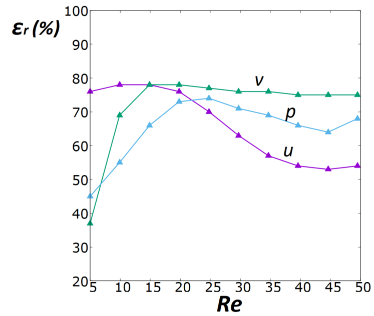


Figure 2. Relative error $\varepsilon_r = 1 - ||x_a - y||/||x_b - y||$ as a function of Re for all velocity components (u, v) and pressure p ($f = 100\%$), using the same ensemble statistics extracted at $Re = 10$.

4. CONCLUSIONS

A localised ensemble-based data assimilation (LEnBLUE) method has been applied to the case of a steady-state, laminar flow around a square cylinder. Synthetic measurements were produced from a finer grid solution using the same CFD code and setup and the spatial discretisation step was assumed to be the sole uncertainty parameter for the creation of the ensemble. The computational cost is mitigated by the implementation of explicit localisation.

Application of the method to the three implicated variables (u, v and p) yielded a reduction in the error of the drag coefficient from 5.58 to 0.98 %. Even when only surface pressures (0.08% of the available pressure data) were assimilated along with all velocity measurements, the error was reduced to 1.24%. In a novel attempt, the method was also successfully applied to the case where DA was performed at an order of magnitude range of Reynolds numbers using ensemble statistics calculated from a single specific Re. Error reduction was above 50% for almost the full range of Re thereby giving rise to the idea that perhaps it would be possible to acquire ensemble statistics (which is generally computationally intensive) to be used at different Re numbers for the same flow regime and geometrical configuration. Extension to turbulent flow is a challenging next step.

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